Toward Aerodynamic Optimization of Complex Configurations

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Motivation

- Aerodynamic Development Typically “cut&try”
  - Slow (design time doing detailed design iterations)
  - Expensive
  - Relies on physical insight of designer for changes
- Automatic design to reduce time in detail design phase
  - Improved performance
  - Decreased costs

- The progress of the design procedure is measured in terms of a cost function I, representative of some appropriate aerodynamic properties (i.e. drag, target pressure distribution, etc.) which are functions of the flow-field variables (w) and the shape of the boundary $\mathcal{F}$. Then

$$I = I\left(w, \mathcal{F}\right),$$

and a change in $\mathcal{F}$ results in a change of the cost:

$$\delta I = \left[\frac{\partial I^T}{\partial w}\right]_I \delta w + \left[\frac{\partial I^T}{\partial \mathcal{F}}\right]_{II} \delta \mathcal{F}, \tag{1}$$

- Using control theory, the governing equations of the flow field are introduced as a constraint in such a way that the final expression for the gradient does not require multiple flow solutions. This corresponds to eliminating δw from (1).

Reference: Jameson, Martinelli, Alonso, Vassberg, Reuther

Suppose that the governing equation R which expresses the dependence of w and $\mathcal{F}$ within the flow-field domain $\mathcal{D}$ can be written as

$$R\left(w,\mathcal{F}\right)=0. \tag{2}$$

Then δw is determined from the equation

$$\delta R=\left[\frac{\partial R}{\partial w}\right]_I \delta w+\left[\frac{\partial R}{\partial \mathcal{F}}\right]_{II} \delta \mathcal{F}=0. \tag{3}$$

Next, introducing a Lagrange Multiplier ψ, we have

$$\begin{aligned} \delta I &= \frac{\partial I^T}{\partial w}\delta w+\frac{\partial I^T}{\partial \mathcal{F}}\delta \mathcal{F}-\psi^T\left(\left[\frac{\partial R}{\partial w}\right]\delta w+\left[\frac{\partial R}{\partial \mathcal{F}}\right]\delta \mathcal{F}\right) \\ &= \left\{\frac{\partial I^T}{\partial w}-\psi^T\left[\frac{\partial R}{\partial w}\right]\right\}_I \delta w+\left\{\frac{\partial I^T}{\partial \mathcal{F}}-\psi^T\left[\frac{\partial R}{\partial \mathcal{F}}\right]\right\}_{II} \delta \mathcal{F}. \end{aligned} \tag{4}$$

Reference: Jameson, Martinelli, Alonso, Vassberg, Reuther

Choosing ψ to satisfy the adjoint equation

$$\left[\frac{\partial R}{\partial w}\right]^T \psi = \frac{\partial I}{\partial w} \tag{5}$$

the first term is eliminated, and we find that

$$\delta I = \mathcal{G} \delta \mathcal{F}, \tag{6}$$

where

$$\mathcal{G} = \frac{\partial I^T}{\partial \mathcal{F}} - \psi^T \left[\frac{\partial R}{\partial \mathcal{F}}\right].$$

$\boxed{\text{An improvement can be made}}$ with a shape change

$$\delta \mathcal{F} = -\lambda \mathcal{G}^{\mathrm{T}}$$

where λ is small and positive. The variation in the cost function then becomes

$$\delta I = -\lambda \mathcal{G}^T \mathcal{G} < 0.$$

The process is repeated to follow a path of steepest descent until a minimum is reached.

Reference: Jameson, Martinelli, Alonso, Vassberg, Reuther

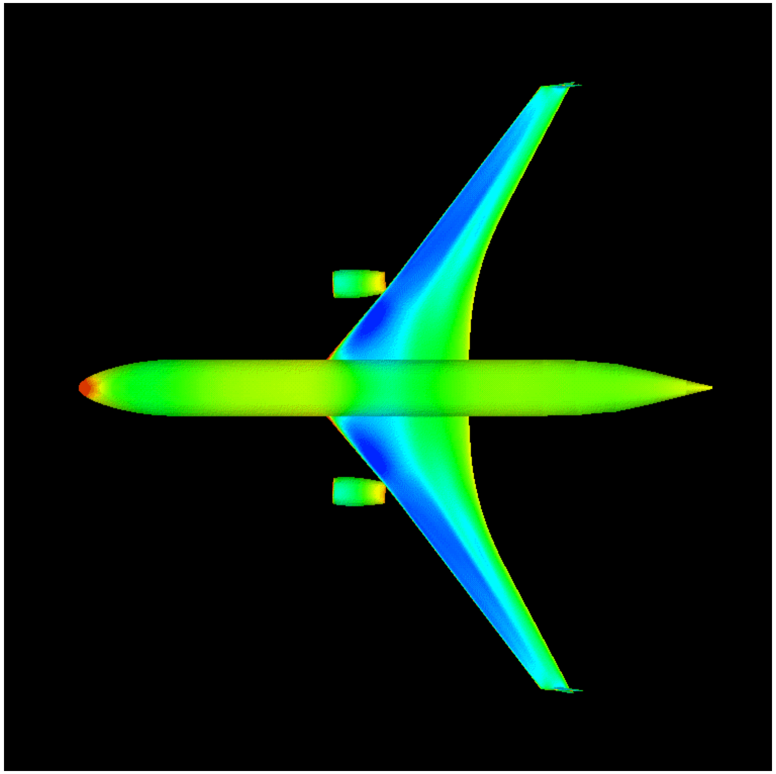
- Equation (6) is independent of δw, with the result that the gradient of I with respect to an arbitrary number of design variables can be determined without the need for additional flow-field evaluations.
- In the case that (2) is a partial differential equation, the adjoint equation (5) is also a partial differential equation. Thus the computational cost of a single design cycle is roughly equivalent to the cost of two flow solutions since the the adjoint problem has similar complexity.
- When the number of design variables becomes large, the computational efficiency of the control theory approach over traditional approach, which requires direct evaluation of the gradients by individually varying each design variable and recomputing the flow field, becomes compelling.

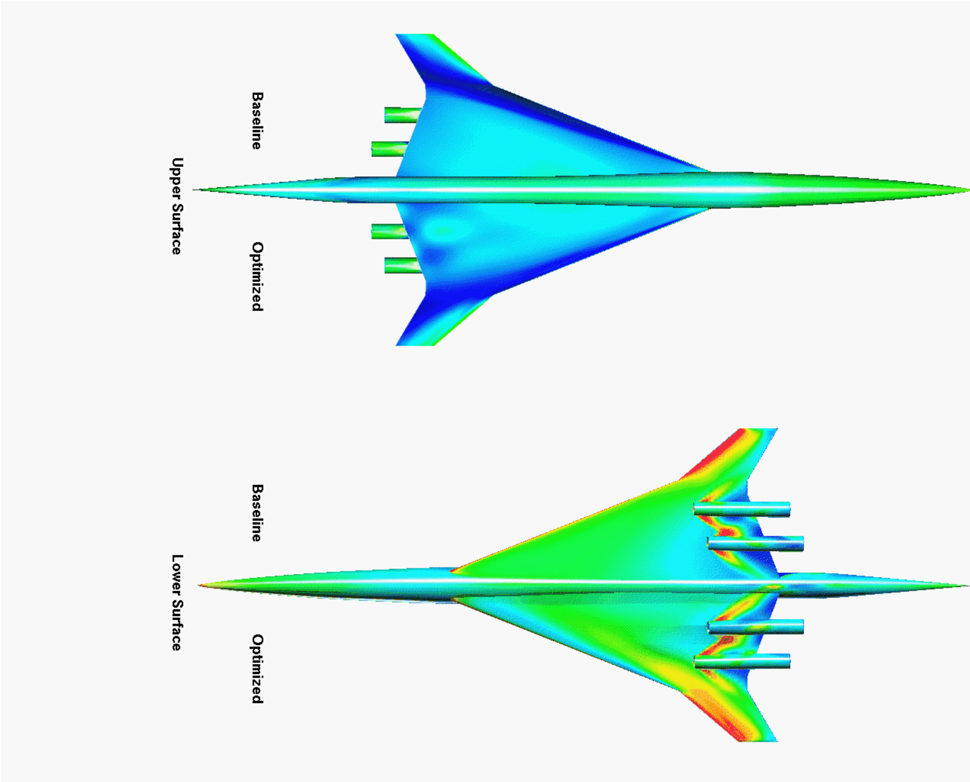
Reference: Jameson, Martinelli, Alonso, Vassberg, Reuther

Design Procedure

1. Solve the flow equations for **r**, u_1, u_2, u_3, p
2. Solve the adjoint equations for y subject to appropriate boundary conditions
3. Evaluate the gradient G
4. Project G into an allowable subspace that satisfies any geometric constraints
5. Update the shape based on the direction of steepest descent
6. Return to step 1 until convergence is reached

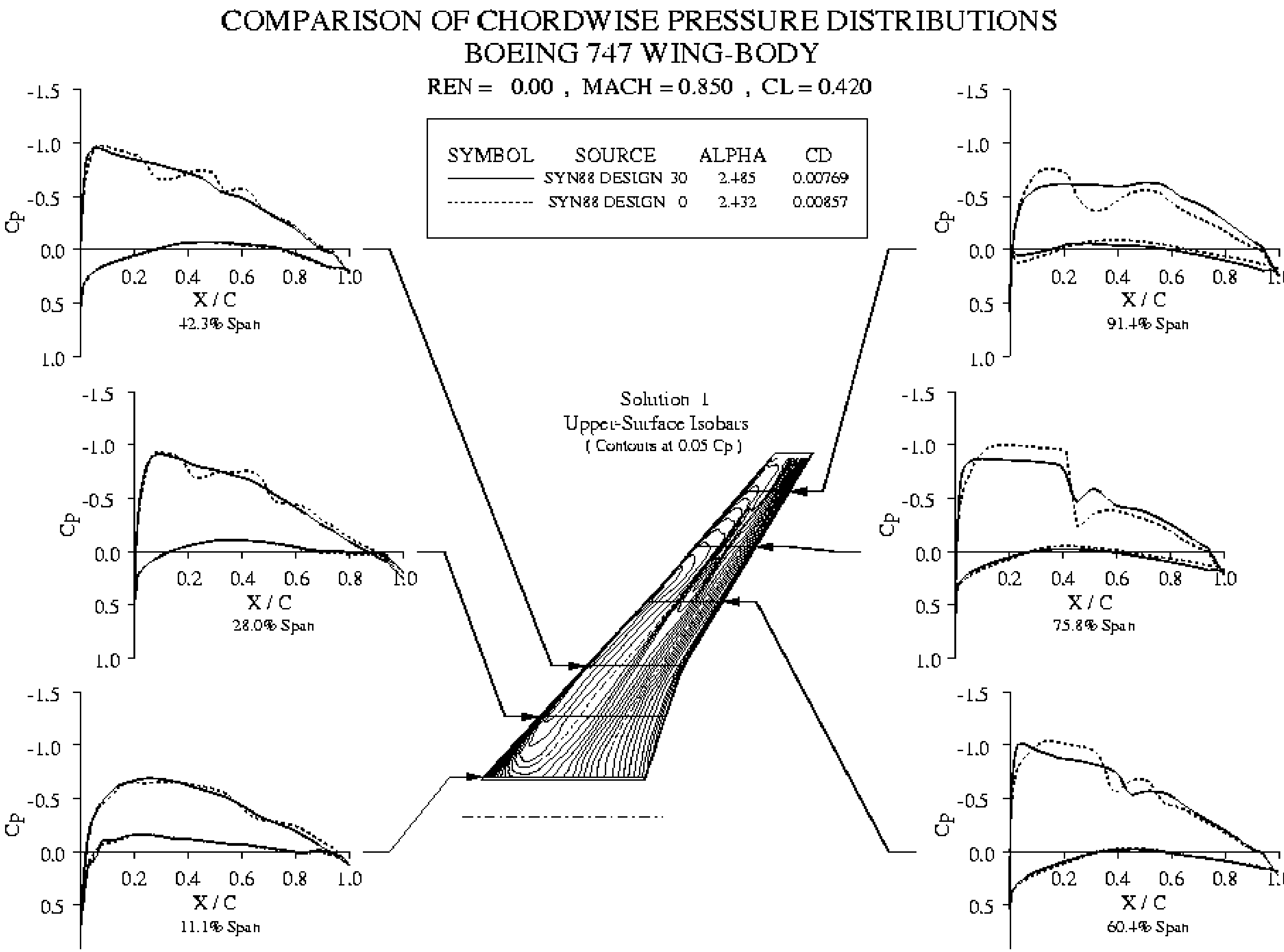
Adjoint Methods Applied Successfully to Optimize Transonic and Supersonic Configurations



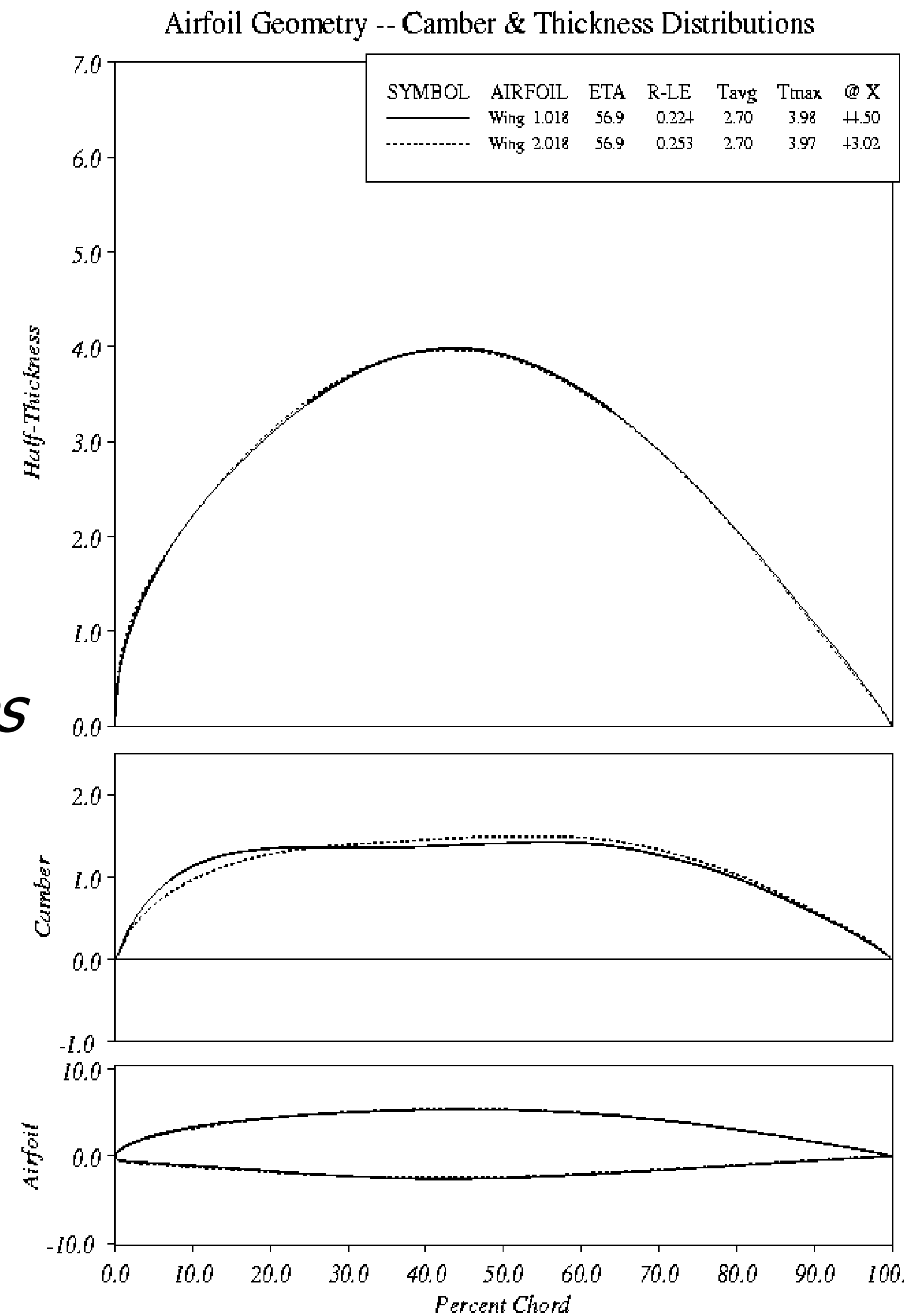


Reference: Martinelli, Reuther, Alonso, Rimlinger, Jameson

Automatic Redesign of 747 – Wing/Body



Reference: Jameson, Martinelli

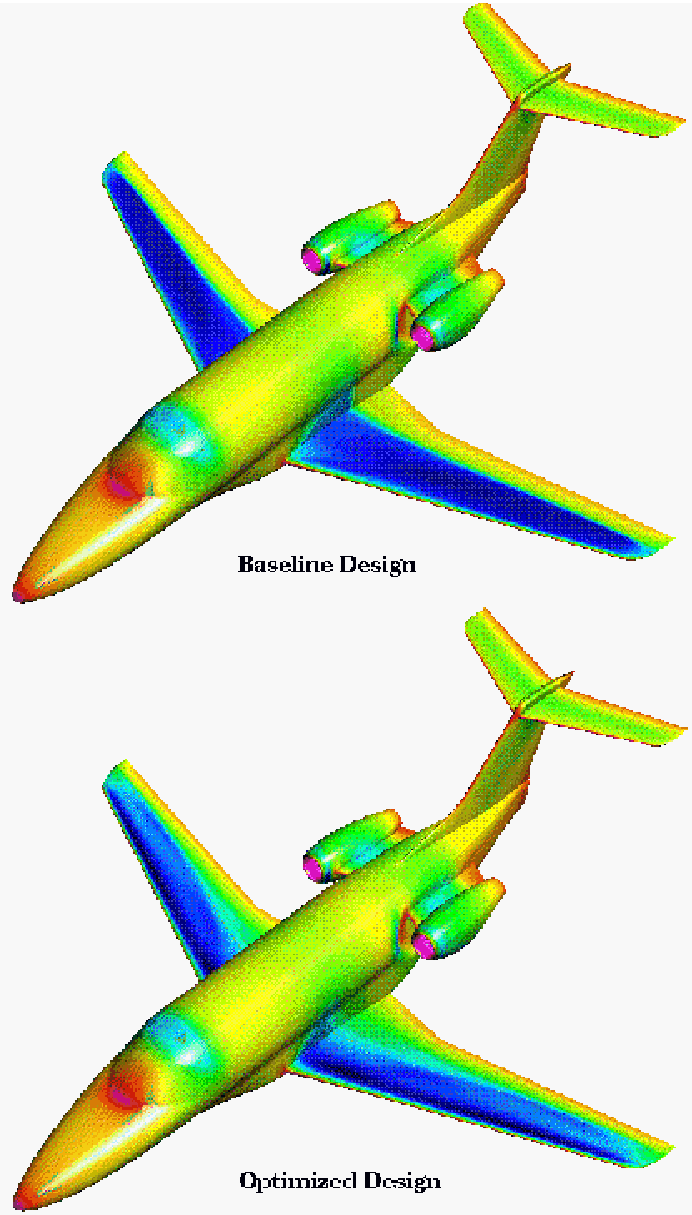


Transonic Performance affected by small changes

Reference: Jameson, Martinelli

Adjoint based optimization has been applied to full configurations

Transonic Multipoint Design
M=0.82, C_L=0.30 shown

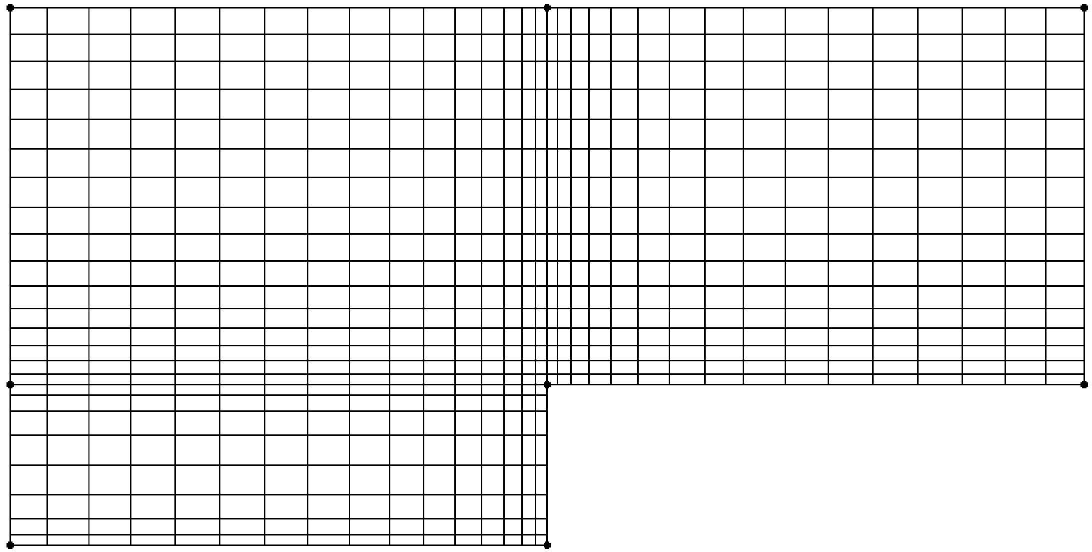


Reference: Jameson, Martinelli, Alonso, Vassberg, Reuther

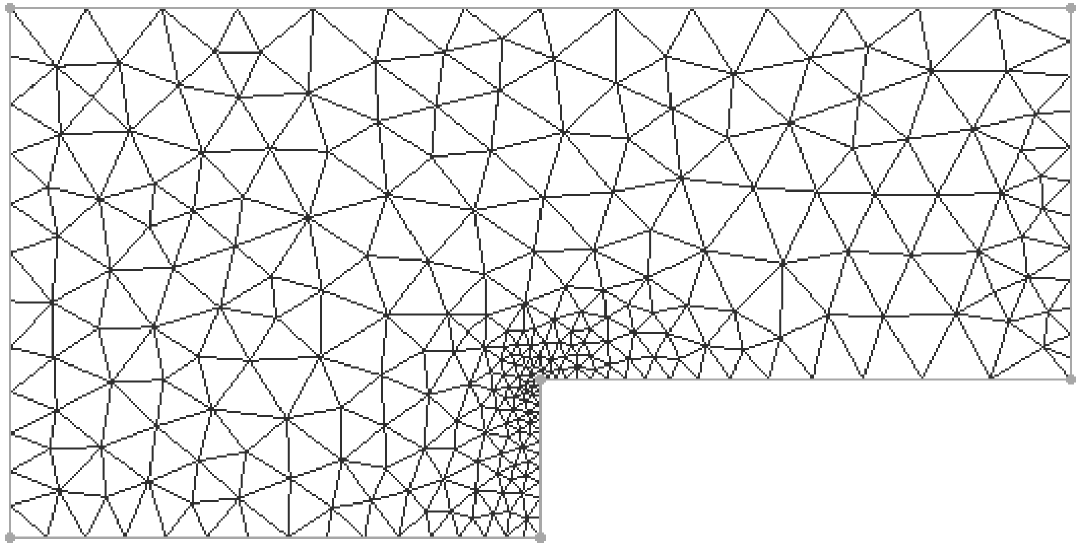
Mesh Type Comparison

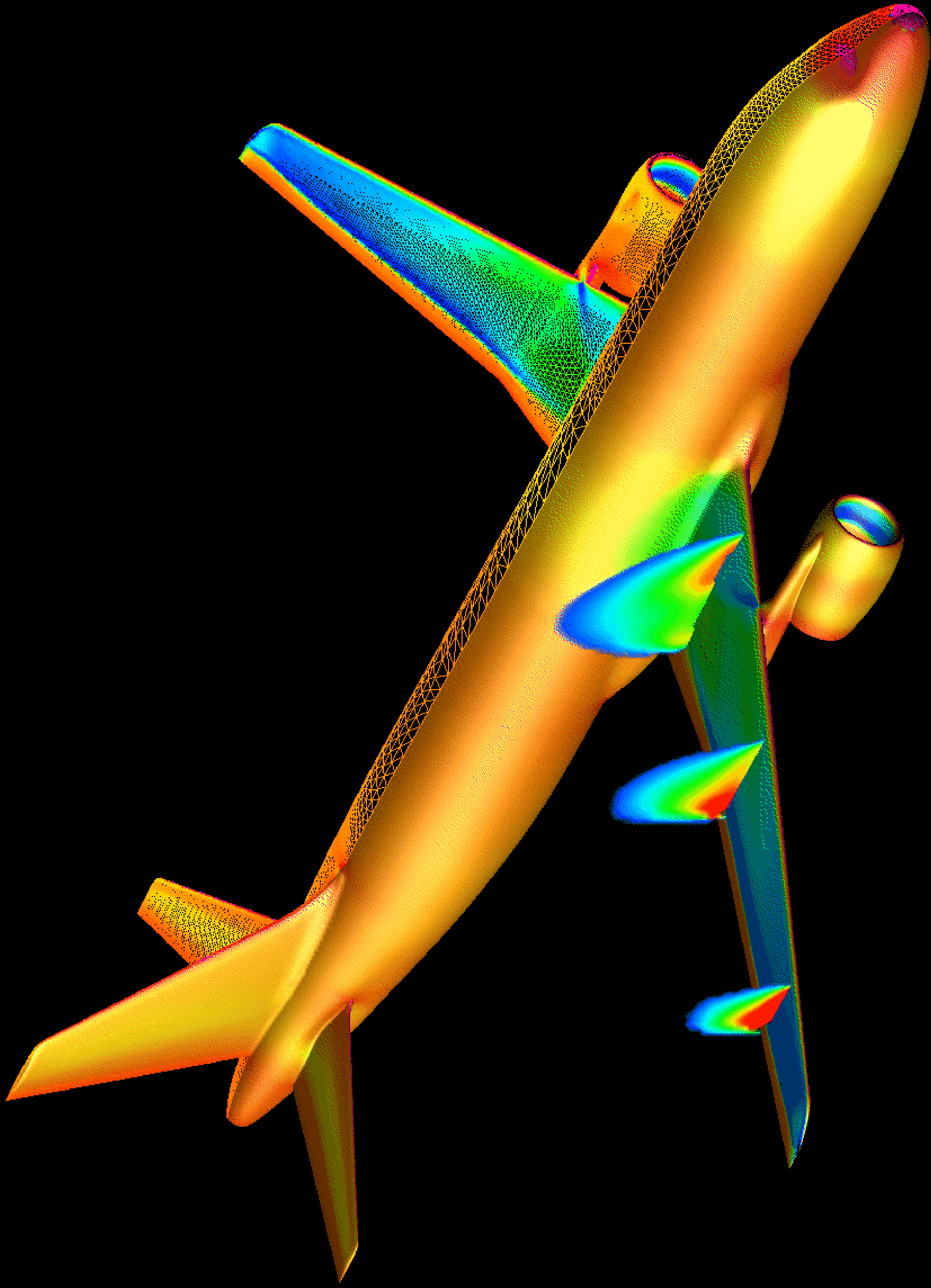
- Structured
  - More developed flow solvers
  - Greater computational efficiency
- Unstructured
  - Ease of construction for complex configurations
  - Efficiency of point placement

Multiblock Structured



Unstructured





Work Plan

- Flow & adjoint solvers are in place (Jameson & Martinelli)
- Write gradient formulation
- Shape modification
  - More general than point movement
  - Integration with CAD

Single Block Structured Mesh